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AUTHOR(S): S. P. Girrens, C. A. Anderson, J. G. Bennett, M. Kramer

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NUMERICAL PREDICTION OF SUBSIDENCE WITH COUPLED GEOMECHANICAL-HYDROLOGICAL MODELING

S. P. Girrens, C. A. Anderson, J. G. Bennett, M. Kramer
Los Alamos National Laboratory
Los Alamos, New Mexico 87545

ABSTRACT

A coupled finite element geomechanical-hydrology code is currently under development for application to the problem of predicting groundwater disturbances associated with mine subsidence. The structural-fluid coupling is addressed by calculating the subsided mine geometry, with emphasis placed on determining the strata disturbance and locating damaged regions, for input into a hydrology code, which determines localized volume flow rates and aquifer fluctuations. Benefits from coupling will be best realized when field measurements, an additional aspect of the study concurrent with analytical investigations, indicating the relationship between increasing rock strain and increasing permeability are incorporated into hydraulic material descriptions. Hydrologic and structural calculations are presented to demonstrate computational capabilities applicable to mine subsidence.

INTRODUCTION

In many mining operations, flooding can cause considerable support problems and may require expensive drainage operations or closing of the mine. Seepage of water into the mine is usually a result of disruption of overlying aquifers or fissures reaching saturated strata or even surface water bodies. In high extraction mining where the overburden is allowed to collapse behind the advancing face, for both legal and practical considerations, the mine operator should be concerned whether his activities might disrupt overlying water supplies. The ability to predict subsurface disturbance on water bearing strata would allow the mine operator to maximize production from reserves which might otherwise be considered too hazardous or questionable to mine.

At the Los Alamos National Laboratory, a study is currently underway to develop a numerical scheme for predicting groundwater movement associated with subsidence. At present, the goal of the program is to provide the mining industry with a practical tool, based upon in situ measurable inputs, to analyze mining hydrological problem . Effort is ongoing in three major areas. The first area is in generating a hydrologic model, which will handle the various groundwater flow phenomena associated with a subsidence geometry. A second area is in developing a subsidence model, though based on elastoplastic principles, that utilizes an additional material model to describe the behavior of geologic media in damaged regions. Notable efforts in modeling damaged regions include those of the Sandia National Laboratory which have had good success in modeling centrifuge experiments.[1-3] However, since the hydrologic calculation is dependent upon the subsidence geometry, effort is being directed at coupling the two calculations to maintain a logical and consistent damage event sequence. Admitting that numerical computations are only as good as the input, the third field of study involves incorporating actual field measurements into the various models, performing the calculations, and then comparing results with measured post subsidence disturbances. This paper presents accomplishments to date and discusses the current scope of work for analyzing the complex problem of subsidence hydrology.

HYDROLOGIC MODELING

In order to study groundwater movement associated with mine subsidence, a highly adaptable hydrologic model is required. The hydrologic model should have the capability of solving three-dimensional, anisotropic, multilayered, confined- and unconfined-aquifer systems. Finite element techniques are well suited for modeling complicated groundwater flow problems. In addition to allowing the investigator sufficient leeway to incorporate new and necessary computational capabilities, for example, infinite elements for bounding large problems, nonlinear flow equations and property descriptions, localized volume flow rate calculations, etc., the method allows coupling with counterpart geomechanics models that deal with subsidence induced deformation.

A derived equation that satisfactorily describes three-dimensional, nonsteady flow in both confined and unconfined aquifers is based on the principle of conservation of mass and Darcy's law for flow in saturated porous media. The principle of conservation of mass for a volume element requires that the mass inflow rate equal the mass outflow rate plus the change of mass storage in time. For a homogeneous and incompressible fluid, this principle is written mathematically as[4]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = - s \frac{\partial}{\partial t} \left(\frac{p}{\rho g}\right) \tag{1}$$

where

- s = specific storage, L^{-1}
- ρ = density of fluid, ML^{-3}
- g = gravity field strength, LT^{-2}
- p = fluid pressure, $ML^{-1}T^{-2}$
- t = time, T
- u,v,w = fluid velocities in the x, y and z directions, LT^{-1}.

The traditional approach to flow in porous media is to express the governing equations in terms of head. Introducing the piezometric head

$$h = \frac{p}{\rho g} + Z \tag{2}$$

where Z = elevation above a given datum, L, Darcy's law relating fluid velocities and pressure gradients may be written

$$u = -K_x \frac{\partial h}{\partial x}, \quad v = -K_y \frac{\partial h}{\partial y}, \quad w = -K_z \frac{\partial h}{\partial z} \tag{3}$$

with K_x, K_y, K_z = the hydraulic conductivity of the saturated flow in the x, y, and z directions, LT^{-1}. Combining Eqs. (1) and (3) we obtain the following expression describing goundwater flow

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) = s\,\frac{\partial h}{\partial t}\,. \tag{4}$$

The concept of hydraulic head thus leads to a mathematical expression involving only one dependent variable, h. Such expressions have been extensively investigated in several fields of mathematical physics, for example, potential theory, diffusion, and heat transfer. Taking advantage of this physical analogy allows the use of an existing three-dimensional, linear and nonlinear steady-state and transient heat transfer analysis code, ADINAT[5]. Since this code is well documented and supported at Los Alamos, the problem of developing a "new" hydrology code becomes one of making the appropriate theory and coding changes, specific to porous flow phenomena, to an established finite element program.

The finite element technique basically involves approximating the boundary value problem by a set of algebraic equations that can be derived by several methods. One technique is to use the method of Weighted Residuals and the Galerkin approximation when applied to Eq. (4) gives the following finite element equation

$$\int_V \left\{[B][K][B]^T\{h\} + \{N\}[S]\{N\}\{\dot{h}\}\right\} dV - \int_S \{N^*\} q_n \, dS = 0\,, \tag{5}$$

where {N} is the vector of element interpolation functions, [B] is the matrix of its derivatives with respect to spatial variables, {h} and $\{\dot{h}\}$ are the vectors of nodal point head and its time derivative, respectively, {N*} are the element boundary shape functions, q_n are the boundary fluid fluxes normal to the surface and [S] is the specific storage matrix. The details of this procedure as applied to finite elements in hydrology are well described in Ref. 20. Equation (5) is programmed into ADINAT using variable number-nodes isoparametric elements and Gaussian quadrature. In the formulation of Eq. (5) no derivative of the hydraulic conductivity matrix [K] exists, and therefore the formulation is valid for constant or variable, that is, anisotropic or possibly head-dependent, values. A modified Newton-Raphson scheme is incorporated in the finite element solution scheme to accommodate nonlinear material descriptions. Prescribed constant-head and fluid-flux boundary conditions are easily applied. Velocity does not appear explicitly in Eq. (5) but is determined indirectly through the application of Darcy's law, Eq. (3), at the Gauss integration points. Transient solutions may be obtained by selecting either central, forward or backward difference time integration schemes.

When developing a computer code to handle the various complex phenomena associated with subsidence, a few established groundwater flow problems should be addressed and compared with results found in the literature to insure confidence in the general solution technique. The first problem considered was the ability to compute unsteady flow to a well pumping an isolated confined aquifer. In order to verify the numerical results, a problem was solved and compared to the Theis nonequilibrium solution of unsteady radial flow to a well pumping at constant rate in an infinite, homogeneous and isotropic aquifer (Ref. 6). Because of the symmetry of radial flow to a single pumping well, an axisymmetric element mesh was used. The aquifer mesh was bounded at a large distance from the well by prescribing exponentially increasing element size in the radial direction. Exponentially increasing time increments were also utilized to reduce computation time. The computed solutions are compared to the Theis solution in Fig. 1. The numerical calculations gave results comparable to the analytical solution with better accuracy obtained by increasing the number of elements. Of interest here is that an aquifer with a 1000 ft radius was analyzed for a period of over 19 h with only 8 elements and 15 time steps. This fact illustrates the power of numerical computations using the finite element technique.

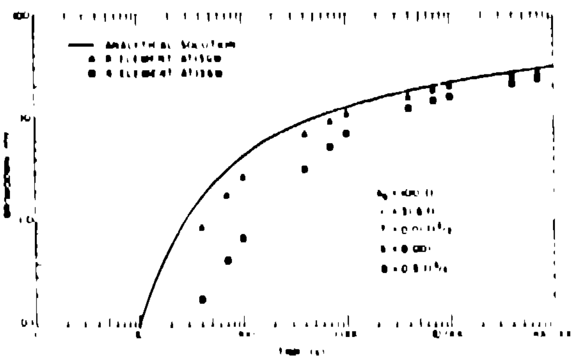


Fig. 1. Comparison of Theis solution with computed results for drawdown in a confined aquifer.

Besides the ability to compute drawdown rates during pumping, another hydrologic computation of interest associated with subsidence might be the capability of calculating leakage through aquitards or semipervious strata. To test this capability, a problem was solved and is illustrated in Fig. 2 where a semipervious layer, overlying a pumped aquifer, has a constant head water table aquifer above itself. The problem is to calculate the unsteady drawdown in the pumped aquifer. Hantush solved the problem analytically assuming: (1) the head in the layer supplying the leakage is constant; and (2) the permeability contrast in the semipervious layer and in the pumped aquifer is very great, so that the flow is vertical in the semiconfining bed and horizontal in the aquifer.[7-9] The discharge of the well is thus supplied by the reduction of storage in the aquifer and by leakage from the semipervious layer. The leakage is obtained from both the reduction of storage in the semipervious bed and from the body of water overlying the semipervious bed. Computed solutions at two different radii are compared with the Hantush solution in Figs. 2 and 3. As is evident, both the analytical and numerical solutions obtain the same steady-state drawdown. However, the solutions differ considerably in the unsteady portion of the curves. This discrepancy is undoubtedly due to the difference in the assumptions made to solve the differential equations. Hantush solves a boundary-value problem where flow is purely vertical in the semipervious layer and purely horizontal in the main aquifer. The hydrology code however solves the entire three-dimensional problem using Eq. (4) and is therefore not pre-biased as to flow direction. We believe therefore that the modified ADINAT code is, within the error of the numerical approximation to the exact solution of Eq. (4), the more accurate of the two methods. This result again shows the increased computational capability obtained from a good hydrologic code.

When dealing with subsidence, one must consider effects upon surface water bodies and water table aquifers. Unconfined aquifers or water table aquifers are characterized by the fact that they possess a free surface. Until recently, the method of analysis for free surface flow was to assume a free surface, discretize the domain below the free surface using finite elements, solve the flow conditions, and then iterate upon the free surface location until the free surface boundary conditions were satisfied.[10] However, with the aid of a nonlinear hydraulic

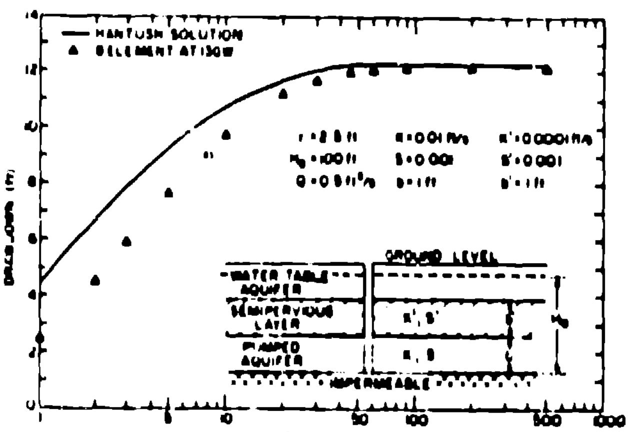


Fig. 2. Comparison of Hantush solution with computed results for a leaky aquifer at r = 2.5 ft.

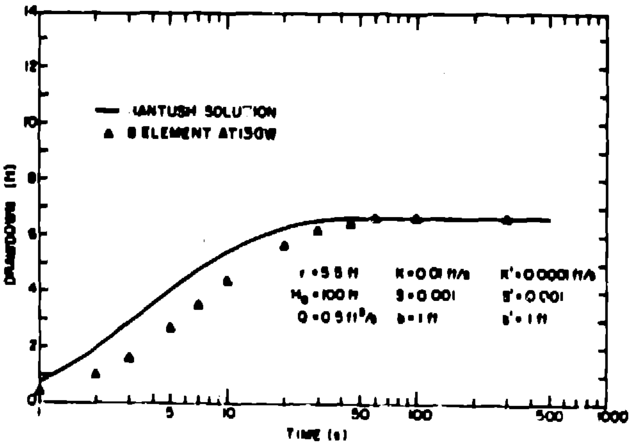


Fig. 3. Comparison of Hantush solution with computed results for a leaky aquifer at r = 5.5 ft.

conductivity description suggested by Bathe,[11] the free surface boundary conditions

$h = z$ and

$\frac{\partial h}{\partial n} = 0$ on the free surface,

where

h = total head above datum
z = elevation above datum
n = direction normal to the free surface,

are satisfied (in an integrated sense) without mesh iteration by the following

$$\text{hydraulic conductivity} = \begin{cases} K \text{ for } h \geq z \\ 0 \text{ for } h \leq z \end{cases} .$$

Incorporating the K dependence on h and z into the modified ADINAT code resulted in the solution shown in Fig. 4 of steady free-surface flow through a porous rectangular dam. The isotropic hydraulic conductivity used in this problem was $K_x = K_y = 1$ ft/h. Figure 5 is a plot of the same problem only showing the velocity vectors computed from the pressure gradients at the Gauss integration points. The free surface is again clearly evident because above it the velocities are zero. The size of the vector drawn is proportional to the velocity magnitude. Such plots are beneficial because they clearly illustrate the relative flow velocities. Once the velocities are computed, determining the volume flow rate across a surface is easily accomplished. To verify this calculational procedure, the volume flow rate coming into the dam shown in Figs. 4 and 5 was computed and compared with the volume flow rate calculated to be exiting the dam. The fluid ingress calculated was 7.96 ft^3/h and the fluid egress along the surface of seepage was 8.08 ft^3/h (the exact answer being clearly 8.00 ft^3/h).

At present, development of the hydrology code is proceeding in several areas to add necessary capabilities, for example, infiltration above

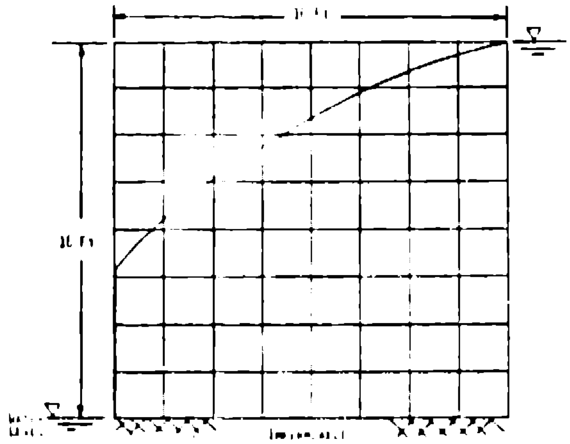


Fig. 4. Computational finite element mesh and calculated free surface contour (h-z=0) through a porous rectangular dam.

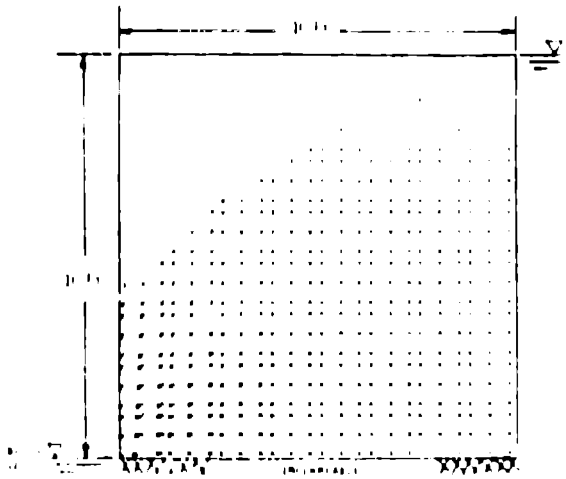
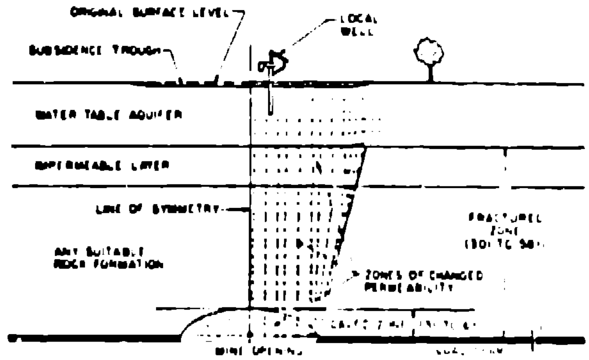


Fig. 5. Velocity vector plot for steady-state flow through a porous dam.

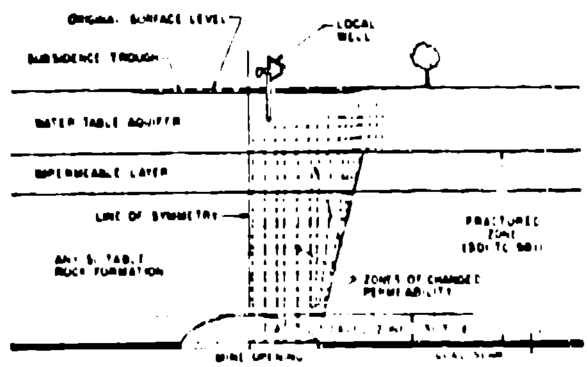
the free surface, variable head boundaries to solve unsteady unconfined well drawdown and non-Darcy flow descriptions for fractured flow. Of prime interest is developing a material permeability flow description relating hydraulic conductivity as a function of rock strain. This relationship will require extensive field data to verify, but the numerical procedures can be developed presently.

An example of how the model can be applied to subsidence problems is shown in Fig. 6. Here, the strata damage discussed by Singh and Kendorvski[17] for a long-wall mine is used in calculations of hydrologic disruption. In the absence of actual permeability measurements, the permeabilities were artificially varied to decrease flow resistance in the areas of greater geologic damage. As is shown by the flow field plots at increasing times, the aquifer water table continues to drop in the vicinity below the surface subsidence trough. The rock formations outside the subsidence draw angle were assumed unaffected and therefore not modeled.

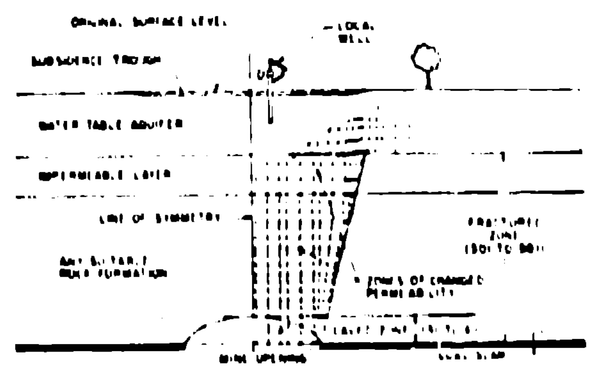
Though relatively simple, this calculation demonstrates what one might expect from the hydrology code coupled with the geomechanical code describing a mine subsidence. Ideally, the structural computation would provide the deformed computational mesh, (geometry for hydrology model) strains, (to be used in hydrology model to adjust permeabilities) and areas of extensive caving or fracturing, (required by hydrology model to decide whether Darcy or non-Darcy flow description is required). The hydrology model would then provide volume flow rates in areas of



(a) Time = 1 day.



(b) Time = 2 days.



(c) Time = 3 days.

Fig. 6. Possible groundwater flow field in vicinity of mine subsidence.

concern, aquifer fluctuations, and possibly even increased infiltration rates from surface water bodies into the mine level.

COUPLED SUBSIDENCE-HYDROLOGY MODELING

A realistic analysis of the effect of coal mine subsidence on disruption of groundwater flow requires that a subsidence model be coupled computationally to the groundwater flow model described previously. The sudsidence model can itself be very complex and for realistic prediction will incorporate multi-layered materials, joints and planes of weakness, elastic-plastic material behavior and models for several different collapse mechanisms, such as those demonstrated by Burns in these proceedings. Although some notable attempts using numerical methods for subsidence prediction[13] have been carried out, it is clear that this problem has not yet been satisfactorily solved in terms of accurately predicting subsidence profiles and volumes of disrupted material. The best success in the area of numerical modeling has been to back-calculate the rock mechanical properties from the observed subsidence profile. See Ref. 14 for a summary of various analytical methods for predicting subsidence.

The general equations that describe fluid migration in highly deformable media, which can be used in a coupled subsidence-hydrology model, are given in Ref. 15. Basically, the equations consist of the Darcy equations (Eq. 3), the equations of static equilibrium of the solid material including the effects of large geometry change, and an equation that couples the pressure in the fluid to the bulk strain in the solid and the volumetric change in fluid trapped in the solid,

$$-\frac{\phi}{K_f}\frac{\partial p}{\partial t} = (\dot{\epsilon}) + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) \qquad (6)$$

where ϕ is the porosity, K_f is bulk compressibility of the fluid, and (u, v, w) are components of the fluid velocity vector. Equation (6) is an extension of Eq. (1) to account for the bulk strain of the solid structure. The coupling between the fluid flow and structural equations occurs in three places; (1) in Eq. (6) above, (2) in the effect of mechanical strain from structural deformation on the rock permeability[16] occurring in the Darcy equation, and (3) in the effect of groundwater flow on rock mechanical properties.

One method proposed for treating the subsidence-hydrology problem is to model the subsidence phenomena separately using a structural finite element code and then follow that calculation with a solution to the fluid flow equations with permeabilities compensated by the pressure and bulk tensile strain calculated for the solid structure. If the groundwater flow markedly affects the mechanical properties then an iteration procedure between the two calculations can be employed. At Los Alamos such a procedure is currently being used with the ADINA and ADINAT codes. Some preliminary ADINA results on a structural model for the Old Ben Mine are presented later.

The difficulty with the iteration procedure is that it neglects the interaction of fluid pressure changes, bulk mechanical strain effects, and fluid flow into or out of an element; that is, the constitutive laws for solid and fluid behavior are not properly coupled. The correct procedure is to introduce the pore pressure as another dependent variable--in addition to the displacements--and to solve a larger system of discretized equations representing the fully-coupled problem. Work at Los Alamos is now underway to develop such a two-dimensional predictive code using the finite element method and the equations of Ref. 15.

CODE QUALIFICATION STUDIES FOR SUBSIDENCE MODELING

Some preliminary work has been completed using the procedure of sequentially executing fluid flow and structural codes. A companion code to the ADINAT code, that has been modified to perform subsurface hydrology calculations, is the structural code ADINA.[18] This code has many features to recommend it for performing subsidence calculations. First of all, the code has been designed from its conception to be a nonlinear analysis code, rather than a modification of a linear code. Thus, users are offered a user-controlled solution strategy scheme for performing modified Newton or other iteration methods. For mine subsidence calculations, an element "Birth/Death" option allows an excellent description of the mining operation including caving and material bulking. Useful material models not only include the usual elastic-plastic strain-hardening models, but also a curve description model that has been designed to describe the behavior of geologic media using bulk loading/unloading moduli and tensile cracking with stiffness reduction when in situ gravity pressure is exceeded by the maximum principal tensile stress. In addition to these standard features, companion pre- and post-processors have been developed at Los Alamos, and code qualification studies have been carried out for features of the code used in reactor safety and weapons design work. Additional slide-line algorithms to prevent adjacent but separated regions from crossing one another, material models, and new elements[19] have also been added to the code. Also, routines have been modified to take advantage of the parallel-processing features of the Cray computers. Our current effort is directed at qualifying the features and models that are suitable for performing subsidence calculations. As an example, Fig. 7 shows a mesh that represents the Old Ben Mine's long wall demonstration project. To check the physical behavior of the element Birth/Death option and the curve description model, a number of calculations of a code-qualifying nature have been run. These results are compared to either known closed form solutions such as the displacement field of a gravity loaded elastic block, or other solutions such as given by Peng[14] for the stress variation around rectangular mine openings with rounded corners. Predicted physical effects are displayed in graphical form such as the subsidence profile at the top of the mesh shown in Fig. 8 for a particular set of best guess geophysical properties. In

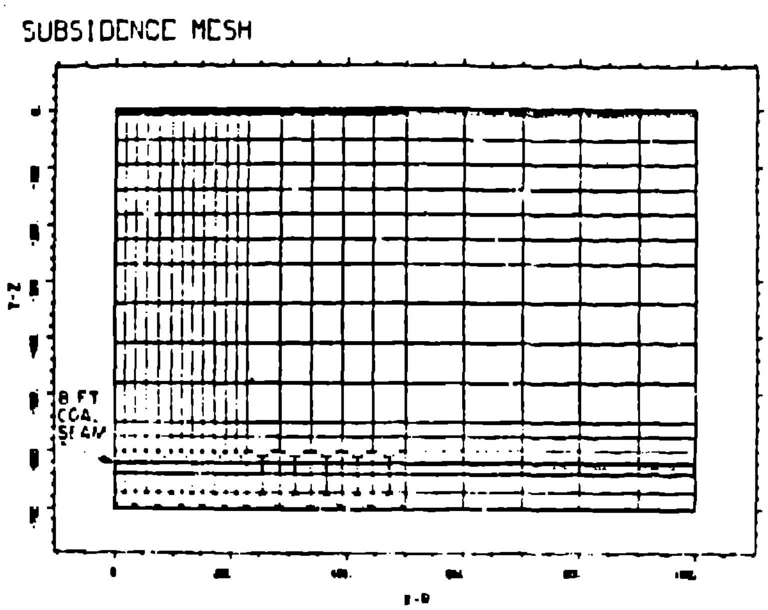


Fig. 7. Computational subsidence mesh for Old Ben longwall demonstration mine.

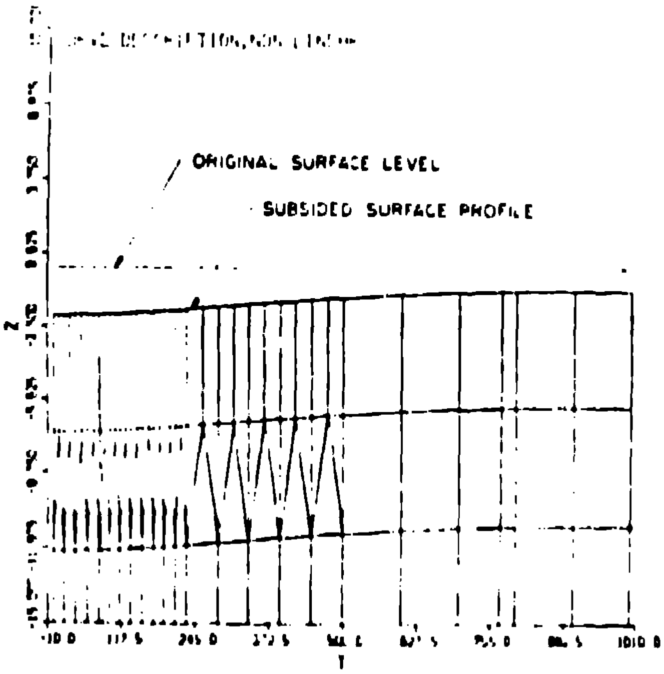


Fig. 8. Deformed mesh illustrating surface subsidence profile for Old Ben longwall mine.

general, the performance of the ADINA code is quite good, given the limitations on "best guess" material properties and the fact that the constitutive equations do not properly describe the material behavior for problems where coupling of the hydrology and subsidence is important.

INCORPORATION OF FIELD MEASUREMENTS AND NUMERICAL MODELS

Because of the large uncertainty associated with predicting groundwater flow and subsidence, it is extremely important that all relevant field measurements be incorporated in the analysis procedure. It is well known, for instance, that the material strength of rock in situ can differ by an order of magnitude from the strength measured on small samples of competent rock.[17] If this is not accounted for in analysis, then the subsidence predicton will be uncertain. The geological formations around the mined area will have uncertainties associated with them as well as the makeup of the hydrological model itself. Much work needs to be done to understand the effect of pressure and bulk mechanical strain on the permeability of overlying rock strata in

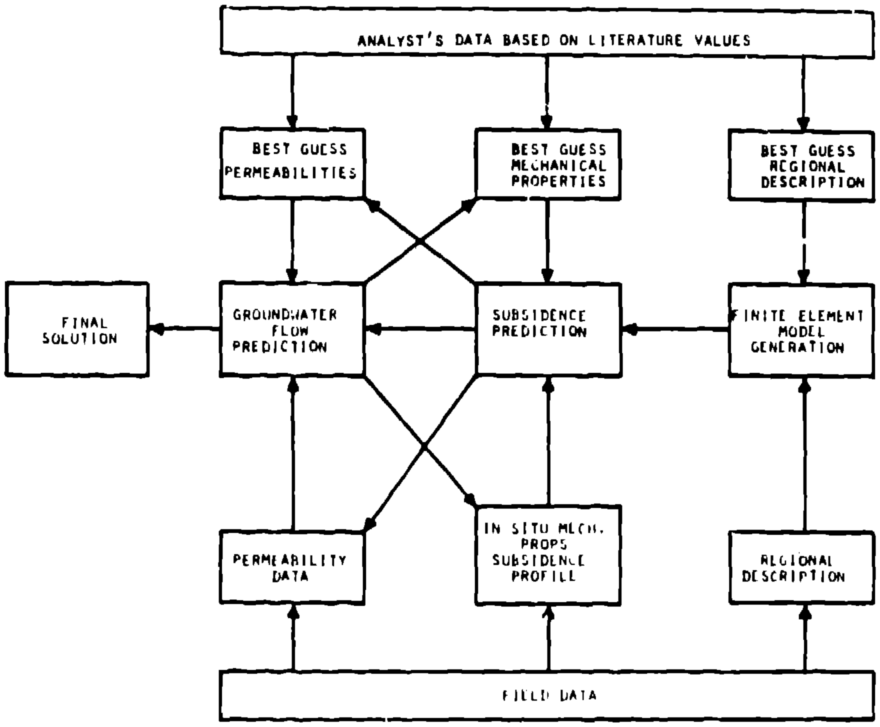


Fig. 9. Flow chart describing interaction of analytical calculations and field measurements for predicting structural-hydrological subsidence effects.

order to firm up the calculation of the groundwater flow after subsidence.

Figure 9 illustrates how an analysis of the groundwater flow around a subsided mine structure might proceed and be made more reliable by the use of field data being gathered during the course of the analysis. At first the analyst has available only best guesses at the regional description, mechanical properties, and permeabilities. Using this he can generate a finite element model of the region.

Initially, all parameter studies with feedback between subsidence and hydrology models and subsequent prediction of groundwater disruptions will take place on the top half of Fig. 9. As a better regional description comes from the field geology, the mesh will be modified. With in situ material property measurements available, better subsidence predictions will become possible and their effects on subsurface hydrology predicted. As the permeability data is established, feedback from the hydrology model will improve the subsidence predictions. Thus the calculational activity moves to the bottom of Fig. 9 until a final solution is predicted. Using this solution, the mine operator will be able to assess the important economic and legal consequences of potential groundwater disruption and subsidence.

We are working closely with DOE consultants to utilize field measurements from test projects at various mine sites. In particular, we are working with GAI Inc. in several hydrologic measurements in the overburden of the Kitt Mine in West Virginia, part of which are described elsewhere in these proceedings. We are also studying several geophysical methods for determining in situ rock mass properties and changes due to subsidence. These include Borehole Jacks, remote sensing data, acoustic tomography and microgravimetry. The latter two are also discussed in these proceedings.

SUMMARY

The problem of predicting groundwater disturbances in the vicinity of mine subsidence is currently being addressed with both numerical analysis and field measurements. For a realistic analysis of the phenomena involved, coupling between a subsidence model and a groundwater flow model is required. Besides adding various basic hydrologic computation capabilities to the groundwater model, work is proceeding to develop a material permeability flow description relating hydraulic conductivity as a function of rock strain. Work on the subsidence model is directed at designing a curve description model describing geologic media behavior using bulk loading/unloading moduli and tensile cracking with stiffness reduction when gravity pressures exceed maximum principal tensile stresses. Field measurements are being obtained concurrent with the analytical developments to verify computational results and establish required material descriptions. Numerical coupling of a groundwater flow model to a geomechanical deformation model is being pursued to provide the mine operator with a complete predictive tool to analyze subsidence effects on actual hydrologic systems.

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